
Equality

■ Properties of Equality

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|----|---------------------------------------|------------|
| 1. | $a = a$ | Reflexive |
| 2. | If $a = b$, then $b = a$ | Symmetric |
| 3. | If $a = b$ and $b = c$, then $a = c$ | Transitive |

■ Equality and Principal of substitution

$a = b$ means that a and b are names for the same object. Therefore, a may replace b (and b may replace a) in any sentence without changing the truth of the sentence.

Axioms

Commutative	$x + y = y + x$	$x \cdot y = y \cdot x$
Associative	$x + (y + z) = (x + y) + z$	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$
Identity elements	$x + 0 = x$	$x \cdot 1 = x$
Inverse elements	$x + (-x) = 0$	$x \cdot \frac{1}{x} = 1, x \neq 0$
Distribution of multiplication over addition		$a(b + c) = ab + ac$

Definitions

◆ Definition. Subtraction

$x - y$ means $x + (-y)$

◆ Definition. Division

$x \div y$ means $\frac{x}{y}$ means $x \cdot \frac{1}{y}$, where $y \neq 0$